MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2019/2020

DEM5028 – ENGINEERING MATHEMATICS 2

(Diploma in Electronic Engineering)

14 MARCH 2020
9.00 AM – 11.00 AM
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **2** pages excluding the cover page with **4** questions only.
2. Attempt **ALL FOUR** questions.
3. Write your answers in the answer booklet provided.
4. Formulae are provided in the appendix section.

QUESTION 1

a. Evaluate $\int_0^3 f(x)dx$ where $f(x)=\begin{cases}2x^2-1 & if \quad x\le 1\\ \dfrac{3}{x}-5+ & if \quad x>1\end{cases}$. (12 marks)

b. Use the **Substitution Rule** to find $\int \frac{9y^2}{\sqrt{3+3y^3}}dy$. (6 marks)

c. Find the following integral

$$\int x^2 e^x dx$$

(7 marks)

[TOTAL 25 MARKS]

QUESTION 2

a. With an assistance of graphs, find the area of region bounded by $y=x^2+3$, $y=-x-1, x=0$ *and* $x=1$.

(9 marks)

b. Sketch and find the volume of the solid obtained by rotating the region bounded by $x=y^2$ and $x=5y$ about the y axis.

(11 marks)

c. By using cylindrical shells, calculate the volume of the solid generated when the region enclosed between $y=\sqrt{2x}$, $x=4$, $x=16$ and the x-axis is revolved about the y-axis.

(5 marks)

[TOTAL 25 MARKS]

Continued ...

QUESTION 3

Solve the differential equation.

a. $\dfrac{dy}{dt} = \dfrac{te^{t}}{y\sqrt{1+y^{2}}}$ (12 marks)

b. $xy' - 2y = x^{2}$ (13 marks)

[TOTAL 25 MARKS]

QUESTION 4

a. Given $a = -3\vec{i} + \vec{j} + 7\vec{k}$ and $b = -5\vec{i} + 3\vec{j} + 9\vec{k}$. Find the angle between the vectors. (8 marks)

b. Given $a = \langle 3,2,4 \rangle$ and $b = \langle 1,-2,-3 \rangle$

i. Find the cross product of $a \times b$. (7 marks)

ii. Find $3a - 2b$. (3 marks)

c. Find the equation of the plane that contains the points $P(1,-6,0)$, $Q(-4,2,-5)$ and $R(-2,4,1)$. (7 marks)

[TOTAL 25 MARKS]

End of Page.

APPENDIX I: Formulae

Integration of common functions

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \qquad \int \frac{1}{x} dx = \ln|x| + C \qquad \int e^x dx = e^x + C \qquad \int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int \sin x dx = -\cos x + C \qquad \int \cos x\, dx = \sin x + C \qquad \int \sec^2 x\, dx = \tan x + C \qquad \int \sec x \tan x dx = \sec x + C$$

$$\int \csc^2 x dx = -\cot x + C \qquad \int \csc x \cot x dx = -\csc x + C$$

Inverse Trigonometry

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$$

$$\int \frac{1}{x^2+1} dx = \tan^{-1} x + C$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \cot^2 x = \csc^2 x$$

$$1 + \tan^2 x = \sec^2 x$$

Integration by parts

$$\int u dv = uv - \int v du$$

Areas Between Curves

$$A = \int_a^b [f(x) - g(x)] dx$$

$$A = \int_c^d [w(y) - v(y)] dy$$

Volume by Washer

$$V = \int_a^b \pi\left(\left[f(x)\right]^2 - \left[g(x)\right]^2\right) dx$$

$$V = \int_c^d \pi\left(\left[w(y)\right]^2 - \left[v(y)\right]^2\right) dy$$

Volume by Cylindrical Shells

$$V = \int_a^b 2\pi x\left(f(x) - g(x)\right) dx$$

$$V = \int_c^d 2\pi y\left(w(y) - v(y)\right) dy$$

Linear Differential Equations:

$$\frac{dy}{dx} + p(x)y = q(x);\ \mu y = \int \mu q(x) dx \Rightarrow y = \frac{1}{\mu}\int \mu q(x) dx, \qquad \text{where } \mu = e^{\int p(x) dx}$$

Divergence Test	If $\lim_{n\to\infty} a_n \neq 0$, then $\sum a_n$ diverges.
p-series	The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if p > 1 and divergent if $p \leq 1$.
Limit Comparison Test	Let $\sum a_n$ and $\sum b_n$ be series with positive terms such that $\lim_{n\to\infty} \frac{a_n}{b_n} = c$ If $0 < c < \infty$, then both series converge or both diverge.
Alternating Series Test	If the alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 + \ldots \qquad b_n > 0$ Satisfies : i. $b_{n+1} \leq b_n$ for all n ii. $\lim_{n\to\infty} b_n = 0$ then the series is convergent
Ratio Test	Let $\sum a_n$ be a series with nonzero terms such that $L = \lim_{n\to\infty} \frac{\lvert a_{n+1}\rvert}{\lvert a_n\rvert}$ a. Series converges absolutely if $L < 1$ b. Series diverges if $L > 1$ or $L = \infty$ c. No conclusion if $L = 1$

Vector

The length of the vector $\mathbf{a} = < a_1, a_2, a_3 >$ is $|a| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

If θ is the angle between the vector **a** and **b**, then $\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$ & $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin\theta$

Cross Product

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} i - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} j + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} k$$

Equation of Line

Vector equation: $r = r_0 + tv$ Parametric equation: $x=x_0+at$, $y=y_0+bt$, $z=z_0+ct$

Equation of Plane $< a, b, c >< x - x_0, y - y_0, z - z_0 >= 0$

The Chain Rule

Suppose that $z = f(x, y)$, where $x = g(t)$ and $y = h(t)$ $\Rightarrow$ $\dfrac{dz}{dt} = \dfrac{\partial f}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial f}{\partial y}\dfrac{dy}{dt}$

Second Derivatives Test

Suppose that $f_x(a,b) = 0$ and $f_y(a,b) = 0$ [that is, (a, b) is a critical point of f]. Let

$D = D(a,b) = f_{xx}(a,b) f_{yy}(a,b) - \left[f_{xy}(a,b)\right]^2$

a. If $D > 0$ and $f_{xx}(a,b) > 0$, then $f(a, b)$ is a local minimum.
b. If $D > 0$ and $f_{xx}(a,b) < 0$, then $f(a, b)$ is a local maximum
c. If $D < 0$, then $f(a, b)$ is a saddle point.

Moments and Centers of Mass

The **moment** about the ***x*-axis:**

$$M_x = \iint_D y\rho(x,y)dA$$

The **moment** about the **y-axis:**

$$M_y = \iint_D x\rho(x,y)dA$$

The coordinates $(\bar{x}, \bar{y})$ of the center of mass:

$$\bar{x} = \frac{M_y}{m} = \frac{1}{m}\iint_D x\rho(x,y)dA \qquad \bar{y} = \frac{M_x}{m} = \frac{1}{m}\iint_D y\rho(x,y)dA$$

Where the mass:

$$m = \iint_D \rho(x,y)dA$$

Triple Integrals: $\iiint_B f(x,y,z)dV = \int_r^s \int_c^d \int_a^b f(x,y,z)dxdydz$